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ROUTING AND SCHEDULING WITHIN A JOB SHOP FRAMEWORK

by

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ABSTRACT

In this paper, an extension of the classical scheduling problem is considered. It is assumed that tasks are processed on parallel processors, available at different production sites. Each task also requires for its processing additional (reusable) resources which, at the beginning, are available at resource sites and production sites. The problem is to find a schedule for processing all the tasks and transporting the resources required between resource and production sites in order to minimize certain objective functions. In the paper, several variations of the problem are considered and mathematical programming formulations for each subproblem are given.

Key words : scheduling, job shop, production planning, flexible manufacturing systems, transportation.

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RÉSUMÉ

On considère dans cet article une extension du problème classique d'ordonnancement de tâches. On suppose que les tâches sont exécutées sur des processeurs parallèles, disponibles à différents sites de production. De plus, chaque tâche requiert pour son exécution, des ressources réutilisables additionnelles qui, au début, sont disponibles à des sites de ressources et aux sites de production. Le problème consiste à déterminer un horaire d'exécution des tâches et un plan de transport des ressources entre les divers sites, afin de minimiser certains objectifs. On considère dans cet article diverses variations du problème ainsi que des formulations par programmation mathématique.

Mots clés : ordonnancement de tâches, job shop, planification de la production, systèmes d'ateliers flexibles, transport.

1. INTRODUCTION

In classical scheduling theory, it is usually assumed that a certain set of tasks is to be processed on a certain set of processors (machines) in order to optimize a given performance measure (see [1,6,8]) for surveys). More recently, a new model has been introduced in which tasks require additional (renewable) resources for their processing in addition to processors. (A summary of the results obtained in this area may be found in [2].) However, it appears that the above model is not always adequate for practical situations. Especially in flexible manufacturing systems, it is sometimes the case that the processing of a given set of tasks is distributed among production sites which contain different numbers of machines [9]. On the other hand, resources are distributed among resource sites (and possibly production sites) and the problem now consists of finding a schedule for transporting resources between resource and production sites, and for processing tasks at production sites, in order to minimize certain objective functions. In this paper, several objective functions are considered, and mathematical programming formulations as well as complexity analysis of the corresponding problems are presented. To begin we define the problems considered more precisely.

We are given a set SR of p **resource sites**. We assume for the time being that there is only one **resource** type, the same at all sites. There is also a set SP of q **production sites**, where site $\ell \in SP$ contains m_ℓ **processors** assumed to be identical. The total number of processors is equal to $m = \sum_{\ell \in SP} m_\ell$. Each site $k \in SR \cup SP$ contains A_k resource units at time 0.

We are also given a set T of n **tasks** to be processed at production sites. Tasks are distributed among the latter and the subset of tasks to be processed at site $\ell \in SP$ is denoted by T_ℓ, where $|T_\ell| = n_\ell$ and $\sum_{\ell=1}^{q} n_\ell = n$. Tasks are **independent** (i.e. no precedence relation among them exists

and **non-preemptable** (i.e. any task once started must be completed without any break). Tasks are also assumed to be of unit length (unit processing time). Task i requires for its processing one processor and a number of resource units specified by its **resource requirement** r_i, i=1,...,n.

In order to fulfil varying resource demands at production sites, resource units may be transported between resource and production sites and between any two production sites. The number of **time** units required to **transport resources** from site $k \in SR \cup SP$ to site $\ell \in SP$ is denoted by $t_{k\ell}$. This time is assumed to be independent of the quantity of resources transported. However, a certain **transportation cost per resource unit,** denoted by $c_{k\ell}$, is associated with the transportation of a resource unit between sites $k \in SR \cup SP$ and $\ell \in SP$.

The problem is now to find a feasible transportation-production schedule which obeys all the above conditions imposed on task performance, resource usage and resource transportation, and which minimizes certain a optimality criterion (objective function). In the following three sections we consider a minimization of transportation cost, schedule length and aggregate cost, respectively.

2. MINIMIZATION OF THE TRANSPORTATION COST

Our first problem will be to find a feasible schedule which minimizes the sum of transportation costs. We will formulate the problem as a linear integer program (LIP). We minimize transportation costs over a certain period of time whose length β is large enough to guarantee the completion of all the tasks, i.e.

$$\beta > \max_{\ell \in SP} (\alpha_\ell) , \qquad (1)$$

where α_ℓ is the smallest possible duration of the production process at site $\ell \in SP$:

$$\alpha_\ell = \left\lceil n_\ell / m_\ell \right\rceil .$$

The choice of the proper value for β is crucial from the viewpoint of the number of variables in the LIP problem. The value of β depends mainly on the number of tasks n and transportation times $t_{k\ell}$. If the latter can be bounded by a polynomial in n, the value β can also be bounded in this way. Otherwise, β depends polynomially on n and a sum of $t_{k\ell}$.

To formulate our problem we will use the following variables :

$x_{k\ell t}$: denotes a number of resource units that leave site $k \in SP \cup SR$ at time t and arrive at site $\ell \in SP$ at time $t+t_{k\ell} < \beta-1$, $k \neq \ell$, $t=0,\ldots,\beta-1$;[1]

x_{kkt} : denotes the number of idle resource units (in storage) at site k during period t+1, $t=0,\ldots,\beta-1$ (this is done at cost $c_{kk}=0$) ;

$$y_{it} = \begin{cases} 1 & \text{if task } i \text{ is executed during period } t+1, \\ 0 & \text{otherwise,} \end{cases}$$

where $i=1,\ldots,n$; $t=0,\ldots,\beta-1$;

$r_{\ell t}$: number of resource units used for production at site ℓ during period t+1 (after production, these resources can still be used), $\ell \in SP$, $t=0,\ldots,\beta-1$.

The problem is then :

$$\text{(P1)} \qquad \text{Minimize} \quad \sum_{\substack{k \in SP \cup SR \\ \ell \in SP \\ t=0,\ldots,\beta-1}} c_{k\ell} x_{k\ell t}$$

$$\text{subject to} \quad \sum_{t=0}^{\beta-1} y_{it} = 1 \qquad (i=1,\ldots,n) \qquad (2)$$

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[1] In the paper we will assume that time period t, $t=1,\ldots,\beta$, lasts from time t-1 to time t.

$$y_{it} \in \{0,1\} \qquad (i=1,\ldots,n;\ t=0,\ldots,\beta-1) \qquad (3)$$

$$\sum_{i \in T_\ell} y_{it} \leq m_\ell \qquad (\ell \in SP;\ t=0,\ldots,\beta-1) \qquad (4)$$

$$\sum_{i \in T_\ell} r_i y_{it} = r_{\ell t} \qquad (\ell \in SP;\ t=0,\ldots,\beta-1) \qquad (5)$$

$$\sum_{\substack{\ell \in SP \\ 0 \leq t \leq \beta - t_{k\ell} - 1}} x_{k\ell t} \leq A_k \qquad (k \in SR) \qquad (6)$$

$$\sum_{\substack{\ell' \in SP \\ t_{\ell\ell'} < \beta - 1}} x_{\ell\ell' 0} + x_{\ell\ell 0} + r_{\ell 0} = A_\ell \qquad (\ell \in SP) \qquad (7)$$

$$\sum_{\substack{k \in SP \cup SR \\ t' = t - t_{k\ell} > 0}} x_{k\ell t'} + x_{\ell,\ell,t-1} + r_{\ell,t-1} = \sum_{\substack{\ell' \in SP \\ t + t_{\ell\ell'} < \beta - 1}} x_{\ell\ell' t} + x_{\ell\ell t} + r_{\ell t} \qquad (\ell \in SP;\ t=1,\ldots,\beta-1) \qquad (8)$$

$$x_{k\ell t} \geq 0 \text{ and integer} \qquad (k, \ell \in SR \cup SP;\ t=0,\ldots,\beta-1) \qquad (9)$$

$$r_{\ell t} \geq 0 \text{ and integer} \qquad (\ell \in SP;\ t=0,\ldots,\beta-1) \qquad (10)$$

The above constraints can be interpreted as follows. Constraints (2) and (3) guarantee the processing of all the tasks. Inequalities (4) ensure that no more tasks than the number of processors are processed at any given moment t. Equations (5) guarantee that the total resource usage at any time does not exceed the available resources. On the other hand, constraints (6) through (8) ensure the proper flow of resources between resource and production sites in time. The flow constraints are illustrated in Figure 1: nodes 1 to p represent resource sites and all other nodes are production site nodes; they are denoted by (site, time t).

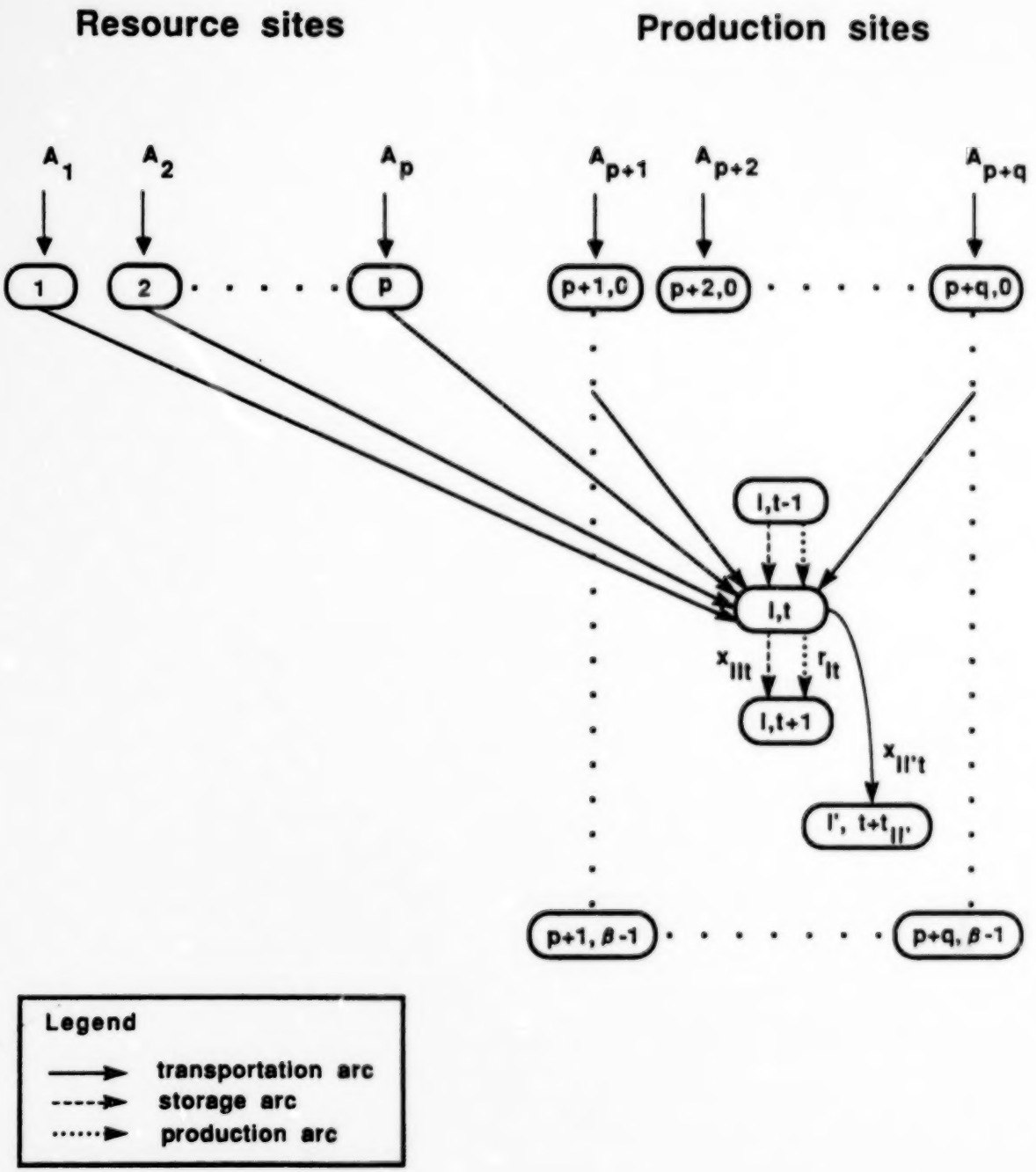

Figure 1 : Network associated with flow constraints.

We now comment briefly on the complexity of the above problem. We see that the number of variables in the problem depends polynomially on the numbers of tasks, processors, resource and production sites and on the value β. The latter may depend pseudopolynomially on the number of tasks. However, even if β is bounded from above by a polynomial in n, the problem may be hard to solve. This is because the subproblem of finding a production schedule only (not taking into account transportation costs) is in general NP-hard [5,3]. The easy exceptions are respectively, two processor and unit resource requirement cases (see [2] for a survey). Thus, in general, our problem is NP-hard and to solve it, one needs an enumerative algorithm. However, if the production schedule is known (i.e. for known y_{it} values), the problem defined by constraints (5) through (10) is a network flow problem. It is unimodular and can therefore be solved in polynomial time by means of a general network flow routine (see for example [7]).

Finally, note that the above formulation can easily be extended to the case where there are several resource types. An additional index would have to be added to variables $x_{k\ell t}$ and $r_{\ell t}$ to indicate the resource type. Constraints (4) through (10) would have to be replicated for all the resource types.

3. MINIMIZATION OF THE SCHEDULE LENGTH

In this section we will consider the problem of finding a production schedule of minimum length, i.e. one which minimizes the completion time of the last task at any of the production sites. In order to formulate the problem in question as a mathematical programming problem, we need a new 0-1 variable :

$$z_t = \begin{cases} 1 & \text{if at least one task is being executed during period t+1 at any one of the production sites,} \\ 0 & \text{otherwise.} \end{cases}$$

We again need an upper bound β on the minimum number of time periods required to execute all tasks. Here the remarks made in the previous section may be useful.

We must now solve the following problem :

$$(P2) \quad \text{Minimize} \quad \sum_{t=0}^{\beta-1} 2^t z_t$$

subject to (2) through (10) and

$$z_t \geq \left(\sum_{i=1}^{n} y_{it}\right)/m \qquad (t=0,\ldots,\beta-1) \qquad (11)$$

$$z_t \in \{0,1\} \qquad (t=0,\ldots,\beta-1) \qquad (12)$$

Two simpler special cases of problem (P2) can be defined as follows :

(P2') Only one production site and only one processor exist in the system. In this case it is not necessary to define z_t. The problem is simply :

$$(P2') \quad \text{Minimize} \quad \sum_{t=0}^{\beta-1} 2^t \left(\sum_{i=1}^{n} y_{it}\right)$$

subject to (2) through (10).

(P2") Only one production site and two processors exist in the system. This subproblem, without a transportation component, can easily be solved by means of the algorithm given in [5]. However, if a transportation component is included, a more elaborate approach is needed. Now, the variable y_{it} can be replaced by a new variable y_{ijt} having the following meaning :

for $i<j$,

$$y_{ijt} = \begin{cases} 1 & \text{if tasks } i \text{ and } j \text{ are both executed during period } t+1, \\ 0 & \text{otherwise} \end{cases}$$

for i=j,

$$y_{ijt} = \begin{cases} 1 & \text{if task } i \text{ alone is executed during period } t+1, \\ 0 & \text{otherwise} \end{cases}$$

Then the problem becomes :

$$\text{(P2")} \quad \text{Minimize} \sum_{t=0}^{\beta-1} 2^t \left(\sum_{i<j} y_{ijt} \right)$$

subject to

$$y_{ijt} \leq r_{\ell t} - r_i - r_j + 1 \qquad (i,j=1,\ldots,n;\ i<j;\ t=0,\ldots,\beta-1) \qquad (13)$$

$$y_{iit} \leq r_{\ell t} - r_i + 1 \qquad (i=1,\ldots,n;\ t=0,\ldots,\beta-1) \qquad (14)$$

$$\sum_{t=0}^{\beta-1} \left(\sum_{i<h} y_{iht} + \sum_{j>h} y_{hjt} \right) = 1 \qquad (h=1,\ldots,n) \qquad (15)$$

$$y_{ijt} \in \{0,1\} \qquad (i,j=1,\ldots,n;\ i<j;\ t=0,\ldots,\beta-1) \qquad (16)$$

and constraints (6) through (10).

In this formulation, constraints (13) and (14) ensure that jobs can only be executed during a given period if the required resources do not exceed the available resources. Constraints (15) specify that all jobs h must be executed. As in (P2) and (P2'), the second factor of the objective function is always equal to 0 or 1, thus ensuring that minimizing any of these objective functions minimizes schedule length. Finally note that index ℓ can be removed from (P2') and (P2") since in these problems, there is only one production site.

4. MINIMIZATION OF THE AGGREGATE COST

In this section, we consider the problem for which the criterion is composed of the two types of costs : the schedule duration cost and transportation costs. As before, we need variables z_t and the upper bound on the schedule length β. A new non-negative parameter γ is introduced which reflects the unit duration cost of a schedule. Then the problem is

$$(P3) \qquad \text{Minimize} \quad \gamma C_{max} + \sum_{\substack{k \in SP \cup SR \\ \ell \in SP \\ t=0,\ldots,\beta-1}} c_{k\ell}\, x_{k\ell t}$$

subject to (2) through (12) and

$$C_{max} \geq t z_t \qquad (t=1,\ldots,\beta-1) \qquad (17)$$

where C_{max} is a minimized value of the schedule length.

Again the above problem is NP-hard in the general case and enumeration algorithms for solving it are required.

5. CONCLUSIONS

In this paper, a new combined model of a production system has been presented involving two stages : a transportation stage and a processing stage. Several criteria have been analyzed and mathematical programming formulations for solving the problems have been proposed. There are still many other problems to be examined in this area. Further research may be concerned with other criteria such as, for example, mean flow times or maximum lateness. More complicated models of task processing involving machines of different speeds and/or machines of different functions should also be considered.

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